anatomical memoir was an investigation upon spermatozoa, published in 1841, with which he took his degree of philosophy at Zürich in 1841, and of medicine at Heidelberg in 1842. In the former year he passed his State examination, of which he records the following contretemps:—

"I, who had at my fingers' ends the finest ramifications of the cranial nerves, the structure of the auditory labyrinth, of the eye, the brain, and so forth, was unable to answer a question on the portal vein."

This is an experience which will assuredly come home to many, and while hence eliciting our sympathies, will at the same time afford no slight consolation to those who reflect on the subsequent achievements of the unfortunate examinee.

In 1841 Kölliker was appointed assistant to Henle, who had received the chair of anatomy at Zürich. In the following year he took a trip to Naples, where he made the acquaintance of Delle Chiaje, Costa and Krohn, and occupied himself with, amongst other things, his well-known studies on the development of Cephalopods. In 1843 he became docent at Zürich, and was prosector to Henle from 1842 until the latter's promotion to Heidelberg in 1844. Henle's chair was then divided into one of anatomy and one of physiology, and Kölliker received the latter; but in 1847 he accepted a call to Würzburg. His departure from Zürich, which was much regretted there, was largely caused by political intrigues in the faculty of the University.

At Würzburg he occupied, at first, the chair of comparative anatomy, but in 1849 he received that of anatomy, which he has now filled for more than fifty years, in a way that needs no praise. The names of many of the most eminent professors of anatomy in Germany, past or present, are to be found in the lists of his pupils or assistants, of whom it is only necessary to mention C. Gegenbaur, Fr. Leydig, R. Wiedersheim, H. Grenacher and Th. Eimer. In 1848 he was associated with von Siebold in founding the *Zeitschrift für wissenschaftliche Zoologie*, of which famous journal he is still one of the editors.

The accounts of his many journeys are compiled, for the most part, from letters written by him at the time to his relations or friends. There is much of interest to be found in them, especially in his visits to England. His first acquaintance with this country was made in 1845, and renewed on many subsequent occasions. In his letters he gives his impressions of England and English life. He quickly made for himself a large circle of intimate scientific friends, amongst whom he mentions, particularly in his earlier letters, the names of Todd, Bowman, Grant, Sharpey, Edward Forbes and Wharton Jones. His time in London seems to have been very well filled up, as he writes in one letter that in the last twelve days he had gone through nine dinners and two breakfasts, some of which do not seem to have been very entertaining. "I took part yesterday in a fearfully wearisome dinner, enough to kill one (etwas ganz totmachendes)," he writes; and further on he complains that "these everlasting dinners, lasting from 6 to 11 o'clock, have taken me *en grippe*, as the French say; but what can one do?" But in other cases he seems to have been happier. In London he is presented at Court, and finds that "the Queen is really pretty, and Prince Albert is also a handsome man." On the eve of his departure, he expresses himself almost as much at home in London, in spite of its size, as in Zürich, and considers it "very interesting, often pleasant, but for the most part fatiguing." He visited this country again in 1850 and 1857, on both of which occasions he spent some, or most, of the time in Scotland, where he became intimately acquainted with John Goodsir and Allen Thomson, and in London with Queckett. His letters from Scotland to C. Th. von Siebold contain some interesting remarks about English science and scientific men.

"The English doctors and physicians are, above all, practical men, and all that pertains to the theoretical side takes with them the second place. This is partly owing to the fact that the English are a people occupied chiefly with commerce, but only partly so; the chief cause of the phenomenon in question is the fact that science does not hold the place it deserves in popular estimation, nor is it supported by the Government in such a way that a man who devotes himself to it can be free from care."

This is the reason, he thinks, why so many men full of enthusiasm for science remain in practice, and finally lose themselves in it; while others regard theoretical studies merely as an advertisement to gain them more clients, since practice in England is golden, and procures for the practitioner a position which contrasts vividly with that of a professor.

"I know only three anatomists and physiologists in England," he adds, "who do not practise—namely, Owen, Sharpey and Grant, of whom Owen alone has a position at all equal to his merits."

In 1850 he also paid a short visit to Oxford, where he met Acland, Strickland and J. V. Carus, but found little that attracted him, and he returned, he tells us, to noisy but infinitely more stimulating London, well satisfied that he was not obliged to spend all his days in "this most peculiar of all university towns."

Space does not permit of reference to the many interesting personal reminiscences or amusing incidents which recur so frequently in this book, especially that detailed in two letters on p. 162, of which we lose nothing by its being to a large extent veiled in the obscurity of the English tongue. It can only be said that the book affords delightful reading, and gives pleasing glimpses of a warm-hearted and charming personality as well as of a great man of science.

E. A. M.

DIFFERENTIAL EQUATIONS.

Theory of Differential Equations. By A. R. Forsyth, Sc.D., F.R.S. Part i. (1890). Pp. xiv+340. Part ii. (1900). Pp. xii + 344, and x + 392. (Cambridge: At the University Press.)

ALTHOUGH these volumes contain more than a thousand pages, it would be premature to express an opinion upon the plan and proportions of Prof. Forsyth's work as a whole; so much of his vast subject still remains unrepresented. Thus the reader will find nothing, except incidentally, of the theory of partial differential equations; and, what is more remarkable, the subject of ordinary linear equations has been reserved for a future volume. However, the two parts which have now been published are so far complete in themselves that it is possible to give some account of their contents, and to appreciate, to some extent, the author's method and point of view.

Part i. treats of exact equations and the problem of Pfaff. Of the two chapters on exact equations it is enough to say that they contain an excellent summary, with well-chosen examples, of the various methods which have been suggested; the most interesting part is that which deals with Mayer's very remarkable extension of Natani's procedure.

The rest of vol. i. is devoted to Pfaff's problem. A chapter on the history of the problem is followed by ten others, which give, in the order of their discovery for the most part, the principal results of Pfaff, Jacobi, Natani, Clebsch, Grassmann, Lie and Frobenius. This plan has its advantages, especially for those who wish to become familiar with the literature of the subject; and mathematical experts will duly appreciate the service which Prof. Forsyth has done them. But if we look at the result as a text-book for mathematical students, it is a question whether the course taken is the best one. A chapter which is an excellent guide to a reader who has before him the original book or memoir upon which it is based, may be simply puzzling to a student unfamiliar with the subject, and unable to refer to the primary sources. It is doubtful, for instance, if any one who has not mastered the *Ausdehnungslehre* will be able to appreciate the chapter on Grassmann's method; and in the same way, the chapters on tangential transformations and Lie's method will not, we fear, do much, in themselves, to arouse an interest in Lie's magnificent discoveries. It is unfortunate that Prof. Forsyth's exclusively analytical attitude has prevented him from utilising Lie's geometrical or quasi-geometrical conceptions. It is quite true that intuitional methods require to be controlled by strict analysis; but they often vivify a mathematical theory in a very instructive and fruitful way. Take, for instance, the question of the "integral equivalent" of the differential relation $Pdx+Qdy+Rdz=0$, where P, Q, R are functions of x, y, z. If we take x, y, z as ordinary Cartesian co-ordinates, this relation associates with any point $A(x, y, z)$ a flat pencil of elementary line-elements, concurrent at A, and lying in a definite plane $P(\xi-x)+Q(\eta-y)+R(\zeta-z)=0$. Thus we may take the "content" of the differential relation to be either a manifold of ∞^4 line elements, or of ∞^3 plane-elements. If the given relation is an "exact equation" $d\phi=0$, the integral $\phi=c$ gives us a family of ∞^1 surfaces, each of which contains ∞^3 line-elements of the content and ∞^2 plane-elements of it. Moreover, every continuous curve made up of line-elements lies (in general) on one of the integral surfaces $\phi=c$, and the line and plane elements of the surfaces exhaust the corresponding elements of the content. These considerations justify us in saying that $\phi=c$, with c an arbitrary constant, is a complete integral equivalent of the differential relation. But in a case like $xdx+zdy-ydz=0$, we cannot construct an integral equivalent of this kind; and the question arises, what integral equivalent, if any, exists, and what will be the nature of its equivalence? To Prof. Forsyth, this is a purely analytical question; he simply inquires what functional relations connecting x, y, z are consistent with the given relation. Of the degree and nature of the equivalence to be expected he says very little; and the gist of what he does say is relegated to a note on p. 250. The geometrical theory at once suggests the possibility of constructing "integral curves" by linking line-elements of the content; a complete integral equivalent may be conceivably constructed by a system of ∞^3 integral curves together exhausting all the line-elements of the content, or again by ∞^2 integral curves, each with ∞^1 associated plane-elements of the content. As an example of the latter kind of integral equivalent, the system of lines

$$x=a, \quad y=bt, \quad z=t$$

where a, b are arbitrary constants, and t is a variable parameter, are integral curves derived from the content of $xdx+zdy-ydz=0$; and if with each point (a, bt, t) we associate the elementary flat-pencil which lies in the plane $a(x-a)+t(y-bz)=0$, we have a complete integral equivalent, all the elements of the content being taken into account. If we take the two analytical relations $x=a$, $y=bz$, involving arbitrary constants only, we get, it is true, a kind of integral equivalent; but this is not complete, in any sense analogous to the complete integral of an exact equation.

Part ii. deals with ordinary equations, not linear; and the point of view is almost entirely that of function-theory. The coefficients in the equation are analytical functions, in Weierstrass's sense; and the main problem is that of discussing the functional nature of the dependent variable or variables. The discussion is necessarily based upon the work of Cauchy, Briot and Bouquet, Weierstrass and Fuchs; the analysis is simple enough in essence, but the details, unfortunately, are unavoidably lengthy, and tend to be monotonous, owing to the necessity of considering different cases and establishing a set of typical forms. The results are so important that the student is bound to make himself familiar with them; but the judicious reader will do well to use his privilege of skipping. The fact is, that the demonstrations fall naturally into a very few types; and it is as profitless to study every one of them minutely as to attempt a detailed examination of every kind of singularity of an algebraic curve. There are, of course, many points in the analysis which cannot fail to arouse interest and admiration; for instance, the use of a dominant function in proving the existence-theorem, and the employment of a sort of extended Puiseux diagram in the applications.

Then, again, there are those surprisingly general and definite results which have been deduced, almost as corollaries, from this somewhat unattractive analytical theory. It must suffice to refer to Painlevé's theorem (ii. p. 211), that the points of indeterminateness of every integral of a single equation of a certain very general type are *fixed points* determined by the differential equation itself; and to the result established by Bruns (iii. p. 311 and following), that every algebraic integral of the differential equation of the problem of three (or more) bodies can be constructed algebraically from the long-known classical integrals. But the reader will find other results of almost equal interest due to Poincaré, Fuchs, Picard and others. The reaction of the Weierstrassian function-theory upon other branches of analysis, and in particular upon the problems of celestial mechanics, is truly remarkable.

It is to be hoped that the publication of Prof. Forsyth's work will make English mathematicians better acquainted with current research on the subjects with which he deals. The value of his treatise for really competent readers is evident, and needs no commendation. But we may, perhaps, regret that he has not more definitely considered the interests of the rising generation. It is most important that new ideas and recent methods should be introduced to young men of ability while their minds are keen and susceptible; and their interest is seldom aroused in the first instance by a treatise which aims at being exhaustive. To take an example in point; few readers, we imagine, to whom the subject was new, would persevere in the study of Lie's great work on transformation-groups; yet what mathematical student could fail to be delighted with his lectures on differential equations with known infinitesimal transformations, as edited by Dr. Scheffers?

No doubt the task of writing an introductory, and thoroughly didactic, treatise on the modern aspects of this theory is very difficult; more so, very likely, than the one to which Prof. Forsyth has applied himself. The selection, combination and assimilation required would demand a great deal of care and judgment; a certain lightness of touch would also be desirable, and this is not easy to maintain after a course of reading in the extremely ponderous memoirs which are so often found in the literature of the subject. But a work of this kind might do more than the most conscientious handbook to encourage a living interest in the theory of differential equations. There is some appearance of a tendency to over-elaboration in English treatises presumably written for students; to authors as well as to lecturers may be commended the maxim "Above all, do not be dull."

G. B. M.

OUR BOOK SHELF.

Origin and Character of the British People. By Nottidge Charles Macnamara. Pp. 242; 33 figures. (London: Smith, Elder and Co., 1900.)

MR. MACNAMARA seeks, in a small compass, to indicate the origin of the component parts of the British people, and to account for the differences of local moral character by proportionate inheritance from the original races, all of which are assumed to have their mental and moral peculiarities as fixed as their physical characters. He believes that the Iberians, as he prefers to call the Mediterranean or Afro-European race, formed the primary stock from which the existing inhabitants of Great Britain and the West of Europe are derived; and that they are the modified descendants of Palæolithic man. The tall fair Aryans originated in Western Asia.

The pioneer migration of the Aryans into Europe formed the Cro-Magnon race; then came the dolmen-builders, the South Mediterranean branch extending from the Amorites to the "fair Libyans"; the migrants into Central Europe mixed with the brachycephals and constituted the "Celts." A distinct northern migration formed the Teutonic Aryans.

The author also believes that dolmens and long barrows are everywhere the work of the Aryan race. The prehistoric tall brachycephals of Northern Europe were a branch of the Northern Mongolian or Turanian race. The short dark brachycephals of Central Europe brought the art of working in bronze from Asia, presumably from Burmah. The Formorians of Ireland were Iberians; in North-west Ireland are still to be found descendants of the Northern Mongoloid race; the Firbolgs were Celtic Aryans or dolmen-builders. The Southern Mongoloids arrived in the bronze age; these are the Tuatha de Danann. A second invasion of Aryan Celts, or Milesians, arrived in Ireland also during the bronze age. This abstract gives a fair idea of the scope and views of the author.

The Geography of the Region about Devil's Lake and the Dalles of the Wisconsin. By Prof. R. D. Salisbury and Mr. W. W. Atwood. Pp. x+151. (Madison, Wisconsin: Geological and Natural History Survey, 1900.)

THIS is the first number of an "Educational Series" to be published by the Wisconsin Geological and Natural History Survey. The region to which attention is now particularly called is in the south-central part of Wisconsin, and it is of interest because it well illustrates many points in the geographical evolution of land-surfaces. It comprises an undulating plain chiefly of Potsdam Sandstone, with some areas of magnesian limestone, and with a northern and southern range of bold quartzite hills. The southern range rises from 500 to 800 feet above the surrounding land, or up to 1600 feet above sea-level, and in the bottom of a deep gap, which divides this range, lies Devil's Lake. This is a lake which, in glacial times, occupied an enclosure between the ice on the one hand and the quartzite ridge on the other: a gorge which originally was the work of a pre-Cambrian stream. The melting of the ice supplied abundant water, and the lake rose perhaps 90 feet above its present level. In this and in many other cases the irregular deposition of glacial drift gave rise to many depressions without outlets, in which surface-waters collected after the ice had disappeared. Few of these lakes now remain in the region, but Devil's Lake, which is more than a mile in length and half a mile wide, occupies an unfilled portion of an old river valley, isolated by great morainic dams from its surface-continuations on either hand. Streams originate beyond these dams. The "Dalles" are sandstone cliffs which form a gorge along the Wisconsin River for a length of about seven miles, and a depth of 50 to 100 feet. The effects of weathering by atmospheric agents, and of erosion by the river, are well exhibited, and the views remind us of the rock-scenery along the Eden near Corby Castle.

The volume, which, with its index, extends to 151 pages, is in reality an essay on the origin of scenery treated from a geological point of view. The authors deal with the pre-Cambrian history of the quartzite, from its origin in loose sand to its uplift and deformation; and they deal similarly with the other strata. They contribute also a fairly full account of the phenomena of the Glacial period, and of the work of rain and rivers. Numerous excellent photographic representations of the scenery are given, including views of various natural arches, tors, and needles.

Monistische Gottes- und Weltanschauung. Von J. Sack. Pp. viii + 278. (Leipzig: Engelmann, 1899.)

IN Herr Sack's view all particular existences are modes of one spirit-substance—God. He calls this doctrine monism, and not pantheism, because he thinks the latter not incompatible with polytheism. Be this as it may, the distinguishing mark of his thesis is that it works to an Hegelian doctrine of being along the lines of a naturalistic theory of becoming that might satisfy Mr.